

# Critical Enhancement of the In-medium Nucleon-Nucleon Cross Section at low Temperatures

T. Alm and G. Röpke

*Arbeitsgruppe der Max-Planck-Gesellschaft "Theoretische Vielteilchenphysik" an der Universität Rostock,  
Universitätsplatz 1, 18051 Rostock, Germany*

M. Schmidt

*Institut für Ostseeforschung, Seestr. 15, 18119 Rostock-Warnemünde, Germany*

The in-medium nucleon-nucleon cross section is calculated starting from the thermodynamic T-matrix at finite temperatures. The corresponding Bethe-Salpeter-equation is solved using a separable representation of the Paris nucleon-nucleon-potential. The energy-dependent in-medium N-N cross section at a given density shows a strong temperature dependence. Especially at low temperatures and low total momenta, the in-medium cross section is strongly modified by in-medium effects. In particular, with decreasing temperature an enhancement near the Fermi energy is observed. This enhancement can be discussed as a precursor of the superfluid phase transition in nuclear matter.

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## I. INTRODUCTION

Heavy-ion reactions at intermediate energies provide information about the equation of state of dense nuclear matter in a broad density and temperature range. Moreover, they can give hints of the process of equilibration of the excited nuclear matter. The time development of the hot source produced in the course of a heavy-ion reaction can be simulated in BUU calculations [1–3]. An important input for such BUU calculations is the nucleon-nucleon (N-N) cross section. The N-N cross sections entering the collisional integral of the BUU equation describe the collision of two nucleons within the hot source, which is formed by the other nucleons produced in the reaction. Consequently, the use of the free N-N cross section is not justified. Instead, the in-medium cross section, that takes into account the modification of the two-particle scattering process within a dense medium, has to be used in such simulations [4].

First results for the in-medium modifications in connection with the mean free path of a nucleon in hot dense matter were obtained by Cugnon et al. [5] and by Schmidt et al. [6]. The in-medium N-N cross section at zero temperature has been calculated by Faessler et al. [7,8] in the framework of Brueckner theory based on the Reid potential. They found strong modifications of the free N-N cross section with increasing density. In particular, their calculations showed a non-monotonous behaviour of the cross section with the density. Ter Haar et al. [9] determined the in-medium cross section in the frame-work of the relativistic Dirac-Brueckner approach at  $T = 0$ , also showing strong deviations from the free cross section. Using the Bonn N-N-potential within the Dirac Brueckner approach Li et al. [10] calculated the in-medium N-N cross section in nuclear matter at zero temperature. In refs. [9], [10] a substantial reduction of the in-medium cross section, particularly for low energies, was found.

Within this paper, we use a thermodynamic Green's function approach with non-relativistic propagators to determine the in-medium N-N cross section at finite temperatures. The calculations have been performed in the ladder approximation for the thermodynamic T-matrix (including Pauli blocking and self energy contributions) that is solved using a separable representation of the Paris N-N-interaction. These methods have already been applied to the calculation of the thermodynamic properties of dense nuclear matter in ref. [6]. We discuss the dependence of the in-medium

N-N cross section on the thermodynamic parameters, i.e. density and temperature, and on the total momentum of the pair. A systematic calculation of the in-medium N-N cross section in a broad parameter range will be given in a separate publication [11] where also the consequences for BUU simulations of heavy-ion reactions will be discussed. Here we will concentrate on the behaviour of the in-medium cross section at low temperatures ( $T < 10$  MeV).

It is wellknown, that in this temperature range a phase transition of nuclear matter into a superfluid state is expected from BCS-theory (see e.g. ref. [12] and references therein). The ladder approximation for the thermodynamical T-matrix gives on one hand side the in-medium cross sections at finite temperatures, on the other hand side it permits the determination of the critical temperature for the superfluid phase transition (Thouless criterion [13]). We consider the relation between the behaviour of the in-medium cross section at low temperatures and the onset of a superfluid state in nuclear matter in detail. In particular, we will show that the temperature at which the in-medium cross section exhibits a divergency (for a given density and zero total momentum of the pair) is identical with the critical temperature found from the Thouless criterion for the onset of superfluidity in symmetric nuclear matter [14].

## II. THE DERIVATION OF THE IN-MEDIUM NUCLEON-NUCLEON CROSS SECTION

The framework for deriving the N-N cross section in the medium at finite temperature will be the Matsubara-Green's function technique [15] (see also [16]) with non-relativistic propagators. The two-nucleon scattering in the medium is described by the thermodynamic T-matrix, which is governed by the corresponding Bethe-Salpeter equation

$$T(121'2') = K(121'2') + \int d3 d3' d4 d4' K(1234) G_1(33') G_1(44') T(3'4'1'2'), \quad (1)$$

1 denotes wavenumber  $k_1$ , spin  $\sigma_1$  and isospin  $\tau_1$  of the nucleons. The ladder approximation for the thermodynamic T-matrix is obtained replacing the four-point interaction  $K$  in (1) by the bare nucleon-nucleon-interaction  $V$ . Within the quasiparticle approximation [15] the product of the two one-particle Greens functions  $G_1$  in (1) yields

$$G_2^0(k_1, k_2, z) = \frac{1 - f(\epsilon(k_1)) - f(\epsilon(k_2))}{z - \epsilon(k_1) - \epsilon(k_2)} \quad (2)$$

where  $z$  is the two-particle energy,  $f(\epsilon)$  is the Fermi distribution function and  $\epsilon(k_1), \epsilon(k_2)$  are the quasiparticle energies. They are defined in terms of the self energy as

$$\epsilon(k_1) = \frac{\hbar^2 k_1^2}{2m} + v(k_1) \quad (3)$$

with

$$v(k_1) = \text{Re}\Sigma(k_1, \omega) |_{\omega=\epsilon(k_1)}, \quad (4)$$

where the self energy  $\Sigma(k_1, \omega)$  was calculated in the T-matrix approximation (for details see ref. [6]).

Within these approximations two medium effects are contained in the quantity  $G_2^0$ . One of these effects is the phase space occupation  $Q$  (Pauli blocking) of the surrounding nucleons given by the  $Q(k_1, k_2) = 1 - f(\epsilon(k_1)) - f(\epsilon(k_2))$  in eq. (2). This form of the Pauli operator takes hole-hole-scattering into account, that is neglected in the usual Brueckner theory taking the Pauli operator as  $Q_B(k_1, k_2) = (1 - f(\epsilon(k_1)))(1 - f(\epsilon(k_2)))$  [15]. The possibility for the Pauli operator to change its sign turns out to be crucial for the onset of superfluidity (see discussion below). The second medium contribution is due to the renormalization of the quasiparticle energies (4) entering eq. (2). In the calculation of the in-medium cross section we used the Paris potential as the bare nucleon-nucleon interaction.

The Paris potential was derived from meson theory and gives a quantitatively reliable description of the on- and off-shell properties of the nucleon-nucleon interaction in the vacuum [17]. In particular, the nucleon-nucleon scattering phase shifts which are well known from experiment are reproduced with a high accuracy. It has been applied to the calculation of the equilibrium properties of nuclear matter as well (see ref. [18]).

After a partial wave decomposition the thermodynamic T-matrix in the channel  $\alpha = (S, L, J)$  reads in Matsubara-Fourier representation

$$T_\alpha^{LL'}(k, k', K, z) = V_\alpha^{LL'}(k, k') + \sum_{k''L''} V_\alpha^{LL''}(k, k'') G_2^0(k'', K, z) T_\alpha^{L''L'}(k'', k', K, z). \quad (5)$$

Relative and center-of-mass coordinates were introduced in eq. (5) according to  $\vec{K} = \vec{k}_1 + \vec{k}_2$  and  $\vec{k} = (\vec{k}_1 - \vec{k}_2)/2$ . In eq. (2) the usual angle averaging of the Pauli operator  $Q(\vec{k}, \vec{K})$  and the quasiparticle energies  $\epsilon(\vec{k}, \vec{K})$  had been carried out. Furthermore an k-dependent effective two-particle mass  $m_{12}^*(k, K)$  was introduced for the evaluation of the quasiparticle energies (see [6] for details). Thus, it is possible to define an effective chemical potential  $\mu_{rel} = \mu - \Delta\epsilon$  relative to the continuum edge, that incorporates the averaged single-particle self energy shift  $\Delta\epsilon$  [14]. Having the T-matrix (5) at our disposal, generalized scattering phase shifts may be defined in the following way (for uncoupled channels) [6]

$$\pi N(E, K, \mu, T) Q(k, K) T_\alpha(k, k, K, E, \mu, T) = \sin \delta_\alpha(E, K, \mu, T) e^{i\delta_\alpha(E, K, \mu, T)}, \quad (6)$$

with the generalized density of states

$$N(E, K, \mu, T) = \frac{km_{12}^*(k, K)}{\hbar^2 2(2\pi)^3}, \quad (7)$$

and the relative energy

$$E = \frac{\hbar^2 k^2}{m_{12}^*(k, K)}. \quad (8)$$

The in-medium scattering phase shifts depend on the temperature  $T$  and the chemical potential  $\mu$  of the medium as well as on the total momentum  $K$  of the pair of nucleons. In the low-density limit  $\mu/T \rightarrow -\infty$  the thermodynamic T-matrix (5) approaches the scattering T-matrix describing the isolated elastic N-N-scattering. Correspondingly, the in-medium scattering phase shifts (6) in this limit approach the free N-N-scattering phase shifts.

Using the partial wave decomposition, which becomes possible after angle averaging of the Pauli operator and the quasiparticle energies, the total thermodynamic T-matrix may be constructed from the partial T-matrices. Suppressing isospin indices and the parametric dependence on  $\mu$ ,  $T$  and  $K$  the partial wave decomposition reads

$$T(\vec{k}, S, M_S, \vec{k}', S', M'_S, z) = \sum_{J, L, L', M_L M_{L'} M_J} T_\alpha^{LL'}(k, k', z) \times (SLM_S M_L | JM_J) Y_L^{M_L}(\hat{k}) (S' L' M_{S'} M_{L'} | JM_J) Y_{L'}^{*M_{L'}}(\hat{k}'). \quad (9)$$

Using eq. (9) the in-medium differential cross section for an unpolarized system is defined via the on-shell T-matrix ( $|\vec{k}| = |\vec{k}'| = k$ ) as

$$\frac{d\sigma}{d\Omega}(k) = \frac{N(k)^2}{(2s_1 + 1)(2s_2 + 1)} \sum_{S, M_S, S', M'_S} \frac{(2\pi)^4}{k^2} |T(\vec{k} S M_S, \vec{k}' S' M'_S)|^2. \quad (10)$$

Integrating eq. (10) over the angle, one arrives at the total cross section in the medium

$$\sigma(k) = \sum_{J,L,L'} \frac{(2J+1)2\pi^3 N(k)^2}{(2s_1+1)(2s_2+1)k^2} |T_{\alpha}^{LL'}(k,k)|^2. \quad (11)$$

Eq. (11) gives the N-N cross section with Pauli blocking in the intermediate states only; i.e. without correction for Pauli blocking in the outgoing channel [9].

For the numerical evaluation of  $T_{\alpha}^{LL'}$  and the cross section (10, 11) we use a separable approximation of the Paris nucleon-nucleon potential [19]. The features of the Paris interaction mentioned above, in particular the reliable description of the empirical two-nucleon scattering data, are preserved in the separable approximation by Plessas et al. [19]. This separable approximation was applied in nuclear matter calculations too [6,12]. The general form of a separable interaction is given by

$$V_{\alpha}^{LL'}(k,k') = \sum_{i,j=1}^N v_{\alpha i}^L(k) \lambda_{\alpha ij} v_{\alpha j}^{L'}(k'), \quad (12)$$

For the detailed form of the form factors  $v_{\alpha i}^L(k)$  and the coupling strength  $\lambda_{\alpha ij}$  for the separable representation of the Paris potential see ref. [19]. The ansatz (12) permits an analytic solution of the T-matrix equation [6] which reads

$$T_{\alpha}^{LL'}(kk'K, z) = \sum_{ijn} v_{\alpha i}^L(k) [1 - J_{\alpha}(K, \mu, T, z)]_{in}^{-1} \lambda_{\alpha nj} v_{\alpha j}^{L'}(k'), \quad (13)$$

with

$$J_{\alpha}(K, \mu, T, z)_{ij} = -4\pi \int \frac{dk k^2}{(2\pi)^3} \sum_{nL} \lambda_{\alpha in} v_{\alpha n}^L(k) v_{\alpha j}^L(k) G_2^0(k, K, z). \quad (14)$$

The total cross section is calculated introducing (13) into eq. (11).

### III. EVALUATION OF THE IN-MEDIUM N-N CROSS SECTION

We will give results for the in-medium total N-N cross section which is expressed by the partial T-matrices (13).

The in-medium total nucleon-nucleon cross section (11) depends on the relative energy  $E$  (or the energy in the laboratory-frame  $E_{\text{LAB}} = 2E$ , where one nucleon is at rest), on the thermodynamic parameters characterizing the medium, namely the density  $n$  (in units of the saturation density  $n_0 = 0.17\text{fm}^{-3}$ ), the temperature  $T$  and via the Pauli blocking  $Q(k, K)$  and the selfenergy shifts on the total momentum  $K$  of the pair. The dependence of the in-medium cross section  $\sigma = \sigma_{np} + \sigma_{nn}$  on the different parameters is illustrated in the following 4 figures. For comparison the free total N-N cross section, as calculated from (11) neglecting all medium effects, was also plotted in Figs. 1-4 (solid line). Taking into account partial waves up to  $L = 2$  we were able to reproduce the experimental free N-N cross section.

In Fig. 1 we present the total nucleon-nucleon cross section as a function of the laboratory energy  $E_{\text{LAB}}$  for various density values at a temperature  $T = 10$  MeV and a total momentum of the pair  $K = 0$ . At a density  $n = 0.1n_0$  the in-medium cross section is still close to the free one showing a slight enhancement. At  $n = 0.5n_0$  we observe a strong suppression of the cross section at very low energies. This is mainly due to the Pauli blocking reducing the available phase space for scattering. In the energy range  $\sim 50\text{MeV} < E_{\text{LAB}} < \sim 150$  MeV a characteristic enhancement with a maximum at  $E_{\text{LAB}} = 80$  MeV occurs. This enhancement will be discussed in detail in the next section. At energies  $E_{\text{LAB}} > \sim 150$  MeV the in-medium cross section is slightly suppressed compared to the free one. The qualitative behaviour of the in-medium cross section for  $n = n_0$  is the same as for  $n = 0.5n_0$ . However, the maximum of the cross section is shifted to higher energies.

In order to study the temperature dependence of the enhancement of the in-medium cross section as given in Fig.1, we plotted in Fig. 2 the total in-medium cross section as a function of  $E_{\text{LAB}}$  for various temperatures at a fixed density  $n = 0.5n_0$  for pairs with total momentum  $K = 0$ . For temperatures higher than  $T = 20$  MeV the cross section approaches the free one and for  $T = 50$  MeV (not indicated) there is only a very slight deviation from the free cross section.

With decreasing temperature the cross section is strongly modified. For temperatures below 20 MeV one observes the following behaviour as a function of the energy:

The cross section is suppressed compared to the free one for low energies  $0 < E_{\text{LAB}} < \sim 50$  MeV and for high energies  $E_{\text{LAB}} > \sim 130$  MeV. Within the energy range  $\sim 50 < E_{\text{LAB}} < \sim 130$  the cross section is enhanced compared to the free one. This enhancement is especially pronounced for low temperatures. For  $T < 10$  MeV a sharp resonance structure develops at  $E_{\text{LAB}} = 90$  MeV (please note that the data in Fig. 2 are plotted on a logarithmic scale). For a critical temperature  $T_c = 4.5$  MeV the in-medium cross section diverges at this particular energy (see discussion in the next section).

In order to demonstrate how this behaviour is changed with the density the in medium cross section is shown in Fig. 3 as in Fig. 2 but for a different density  $n = 0.2n_0$ . The qualitative behaviour of the cross section is the same as in Fig. 2. However, the deviations from the free cross section at a given temperature are smaller for this smaller density value. Again a resonance structure is observed, the maximum of which lies at  $E_{\text{LAB}} = 46$  MeV. Below a critical temperature  $T_c = 4.2$  MeV again a divergence of the in-medium cross section shows up at this energy value.

The in-medium cross section depends on the total momentum  $K$  of the pair via the Pauli blocking. In order to demonstrate how this dependence changes the results given in Figs. 1-3 for  $K = 0$ , we plotted in Fig. 4 the cross section as a function of  $E_{\text{LAB}}$  for several values of the total momentum at a fixed density  $n = 0.5n_0$  and temperature  $T = 5$  MeV. For  $K = 0$  one finds the pronounced resonance ( see Fig. 2) discussed above. With increasing total momentum this resonance is broadened until it disappears for sufficiently high total momenta. For  $K = 400$  MeV/c the in-medium cross section approaches the free cross section. This behaviour results from the fact that at the corresponding higher total momenta  $K$  of the pair the Pauli blocking  $Q(k, K)$  and the medium contributions to the one-particle energies are reduced. Thus, at sufficiently high total momentum  $K$  the medium effects are negligible and the in-medium cross section approaches the free cross section.

#### IV. THE ORIGIN OF THE ENHANCEMENT IN THE IN-MEDIUM CROSS SECTION AT LOW TEMPERATURES

For high temperatures (comparable to the the Fermi energies) the in-medium cross section approaches the free one (see e.g. the  $T = 20$  MeV curve in Fig.2). This is due to the fact that the medium effects (Pauli blocking, selfenergy shifts) are considerably reduced at these temperatures.

The modifications of the in-medium cross section are much more pronounced at low temperatures. The most prominent effect found in our calculations is a peak structure dominating the in-medium cross section at low temperatures. Finally, we found a critical temperature  $T_c$  at which the in-medium cross section at a given density shows a divergency at a particular energy for pairs with zero total momentum  $K = 0$ .

This behaviour may be traced back to the pole structure of the thermodynamic T-matrix (13). Thus, one has to investigate the different channels  $\alpha$  of the thermodynamic T-matrix for the possible occurrence of poles in dependence on temperature and density. With decreasing temperature such a pole first occurs at the particular energy  $E = 2\mu_{\text{rel}}$  ( $\mu_{\text{rel}} = \mu - \Delta\epsilon$ , relative to the continuum edge) and at the critical temperature  $T_c$  in the  $^3S_1 - ^3D_1$  channel.

In order to investigate, how the divergence of the in-medium cross section at  $T_c$  and the resonance like structure in the cross sections at temperatures just above  $T_c$  are related to this pole in the S-D-channel we will use the separable approximation for the T-matrix (13). In ref. [6] an optical theorem for the thermodynamical T-matrix was derived, which is given in momentum representation using eq. (7) as

$$\text{Im}T_\alpha(k, k', E) = T_\alpha(k, k'', E)\pi Q(k'', K)N(E, K)T_\alpha^*(k'', k', E). \quad (15)$$

With the help of this generalized optical theorem (15) for the thermodynamic T-matrix the in-medium cross section can be related to the imaginary part of the thermodynamic T-matrix. For a given partial wave  $\alpha$  (in particular  $\alpha = {}^3S_1 - {}^3D_1$ ) the corresponding partial cross section is given from (11) as

$$\sigma_\alpha(k) \sim |T_\alpha(k, k)|^2 = Q^{-1}(k, K)\text{Im}T_\alpha(k, k), \quad (16)$$

(the indices  $L, L'$  as well as the thermodynamic parameters are omitted), where for a rank one ( $N=1$  in eq. (13)) separable interaction

$$\text{Im}T_\alpha(k, k) = \frac{\lambda_\alpha v_\alpha^2(k)\text{Im}J_\alpha(K, \mu, T, E)}{(1 - \text{Re}J_\alpha)^2 + (\text{Im}J_\alpha)^2} \quad (17)$$

holds. Evaluating the imaginary part of  $J_\alpha$  one finds

$$\begin{aligned} \text{Im}J_\alpha(K, \mu, T, E) &= -g^{-1}(E + K^2 + 2\Delta\epsilon) \int \frac{d^3k'}{(2\pi)^3} f(\epsilon(K/2 + k'))f(\epsilon(K/2 - k')) \\ &\times \lambda_\alpha v_\alpha^2(k')\pi \frac{\delta(k' - k)}{2\hbar^2 k' / m_{12}^*(k, K)}, \end{aligned} \quad (18)$$

with

$$E = \frac{\hbar^2 k^2}{m_{12}^*(k, K)} \quad (19)$$

using the property  $1 - f(\epsilon(k_1)) - f(\epsilon(k_2)) = g^{-1}(\epsilon(k_1) + \epsilon(k_2))f(\epsilon(k_1))f(\epsilon(k_2))$  of the Pauli operator. The quantity  $g(E)$  is the Bose distribution function of the two-particle states. Consequently, at energies  $E = 2\mu_{rel}$  ( $K = 0$ ) the quantities  $\text{Im}J_\alpha$  and  $\text{Im}T_\alpha$  (17) vanish. This zero of  $\text{Im}T_\alpha$  is not restricted to the quasiparticle approximation (2), which can easily be demonstrated by introducing the spectral representation for the full one-particle propagators [20]. In the numerator of eq. (16) this zero is compensated by a corresponding zero from the inverse Pauli operator  $Q^{-1}$  in (16). However, the second term in the denominator of eq. (17) is equal to zero at this particular energy. Consequently, the magnitude of (16) at  $E = 2\mu_{rel}$  is determined by the term  $(1 - \text{Re}J_\alpha(K = 0, \mu, T, E = 2\mu_{rel}))^2$  in the denominator of (17). A pole of the T-matrix  $T_\alpha$  ((13) for a rank one separable ansatz,  $N = 1$ ) occurs where both terms  $\text{Im}J_\alpha$  and  $(1 - \text{Re}J_\alpha)$  are equal to zero. The second condition

$$1 - \text{Re}J_\alpha(K = 0, \mu, T = T_c, E = 2\mu_{rel}) = 0 \quad (20)$$

is fulfilled at the critical temperature  $T_c$ . Thus, the closer the temperature gets to the critical temperature  $T_c$  the larger the partial cross section (16) at  $E = 2\mu_{rel}$  becomes. It then gives the dominant contribution to the total cross section and produces the resonance like structure shown in figs. 2-4. Finally, at  $T = T_c$  the entire denominator of eq. (17) vanishes at  $E = 2\mu_{rel}$  and the corresponding partial cross section diverges.

On the other hand eq. (20) is identical with the Thouless criterion [13]. It states, that the sum of the ladder diagrams (5) converges only above the critical temperature  $T_c$  (20), which is equivalent with the critical temperature

found from BCS theory [13]. Consequently, the critical temperature at which the in-medium cross section for pairs with zero total momentum diverges, coincides with the critical temperature for the onset of superfluidity. In particular, the same condition (20) holds for the onset of superfluidity in symmetric nuclear matter [14] and for the divergence of the in-medium N-N cross section. This result does not depend on the particular choice of a separable interaction [13]. Using a generalized BCS theory for superfluid nuclear matter including pairing in the S-D channel Baldo et al. [21] found a critical temperature  $T_c$  for the onset of superfluidity in nuclear matter in agreement with the calculations of ref. [22] using the Thouless criterion (20). Thus, the sharp resonance like structure in Figs. 2-4 can be interpreted as a precursor for superfluidity.

The position of the resonance or the pole respectively is given according to the Thouless criterion by  $E = 2\mu_{rel}$  ( $E_{LAB} = 2E$ ), where the effective chemical potential  $\mu_{rel}$  (including the quasiparticle shift) is related to the density  $n$  by the equation of state (see [14] for details). The values of twice the effective chemical potential  $2\mu_{rel}$ , corresponding to the densities  $n = 0.5n_0$  (Fig.2) and  $n = 0.2n_0$  (Fig.3), coincide with the peak positions of the resonances as indicated in Figs. 2 and 3.

A relation between two-particle fluctuations above the critical temperature for the onset of superconductivity was discussed by Rickayzen [23]. He found, that the instability of the normal state approaching  $T_c$  from above is signalled by the growth of pair fluctuations  $\langle c_{-k}c_k c_k^+ c_{-k}^+ \rangle$  which tend to infinity at  $T_c$ . Considering the general relation between the two-particle correlation function and the T-matrix it can be shown that these fluctuations are related to the resonance like behaviour discussed above for the in-medium cross section.

The occurrence of a singularity at the transition to a superfluid state has already been obtained by Sjöberg [24] in the calculation of the quasiparticle interaction in nuclear matter.

More recently, the divergence of the pair susceptibility at the critical temperature has been discussed by Schmitt-Rink et al. [25] for a two-dimensional Fermi gas.

The results obtained for the in-medium N-N cross section can be compared with other calculations. Faessler et al. [7], [8] calculated the in-medium neutron-neutron cross section at zero temperature for two colliding nuclear matters from the G-matrix. Although these authors neglected hole-hole scattering and modified the Pauli operator for two colliding Fermi spheres, they obtained a similar resonance like behaviour as shown in Figs. 2-4. The energies of the peaks in refs. [7], [8] agree with the peak energies, indicated as  $4\mu_{rel}$  in Figs. 2 and 3, for the same densities. This corresponds to the findings of Vonderfecht et al. [26] who could show that even neglecting hole-hole scattering in the Pauli-operator a bound pair state in the medium is found in some density range. Within our finite-temperature approach the enhancement of the in-medium cross sections can be related to the onset of a superfluid phase (see discussion above).

Relativistic calculations of the in-medium N-N cross section performed by ter Haar et al. [9] and Li et al. [10] do not show the enhancement of the cross section discussed above. The discrepancy to the non-relativistic calculations of the in-medium N-N cross section in ref. [7] and in this paper as well as to the related mean free path [5], [6] may be due to the different model characteristics.

In conclusion, we would like to summarize our results:

- (i) Whereas for high temperatures (comparable to the Fermi energies) the in-medium cross section approaches the free one, strong deviations from the free cross section were found at high densities and low temperatures and low total momentum. In particular, we found that for a given density there is a strong enhancement of the cross section at low temperature near energies  $E = 2\mu_{rel}$ .
- (ii) Approaching a critical temperature  $T_c$  from above this enhancement leads to a resonance like structure. At

a temperature  $T = T_c$  the in-medium N-N cross section for pairs in the medium with zero total momentum diverges at the particular energy  $E = 2\mu_{rel}$ .

- (iii) Using the Thouless criterion [13] we could show that this divergence happens at the same critical temperature as the onset of a superfluid phase of nuclear matter. Consequently, the resonance structure could be interpreted as a precursor of superfluidity.
- (iv) The strong dependence on the density and the temperature of the surrounding medium is considerably reduced for pairs with non-zero total momentum  $K$ . For high enough total momentum the in-medium cross section approaches the free one.

It should be mentioned in which way the approximations we used could be generalized:

For the evaluation of the T-matrix (1) the ladder approximation was used. Thus, the modification of the N-N interaction in a dense medium (e.g. higher order corrections such as screening) was neglected.

Furthermore our calculation is based on the quasiparticle approximation (2). However, the generalization with a finite width of the one-particle spectral function is in principle straightforward.

It would be interesting to develop a theory which can describe the in-medium N-N scattering below  $T_c$ . This demands approximations going beyond the quasiparticle approximation.

The determination of the in-medium N-N cross section at densities above the saturation densities and at higher energies would require a relativistic treatment such as the Dirac-Brueckner approach as developed in refs. [9] [10].

However, within the standard approximations also used in this paper, the background for the occurrence of an enhanced in-medium N-N cross section at relative energies equal to twice the Fermi energy and temperatures below 20 MeV can be understood as a precursor for the onset of superfluidity. These significant modifications of the in-medium cross section have to be taken into account in simulations of hot expanding nuclear matter as produced in heavy-ion reactions.

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## Figure Captions

Fig. 1:

The in-medium total nucleon-nucleon cross section  $\sigma$  as a function of  $E_{\text{LAB}}$  at given temperature  $T = 10$  MeV and total momentum  $K = 0$  for several values of the density  $n$  (in units of the saturation density  $n_0 = 0.17 \text{ fm}^{-3}$ ). The solid line gives the total free cross section.

Fig. 2:

The in-medium total nucleon-nucleon cross section  $\sigma$  as a function of  $E_{\text{LAB}}$  at a given density  $n = 0.5n_0$  and total momentum  $K = 0$  for several values of the temperature  $T$ . The solid line gives the total free cross section.  $4\mu_{\text{rel}}$  denotes the position of the effective chemical potential as defined in the text. The critical temperature at which the in-medium cross section diverges is  $T_c = 4.5$  MeV.

Fig. 3:

The same as in Fig. 2 for a density  $n = 0.2n_0$ . The critical temperature at which the in-medium cross section diverges is  $T_c = 4.2$  MeV.

Fig. 4:

The in-medium total nucleon-nucleon cross section  $\sigma$  as a function of  $E_{\text{LAB}}$  at given density  $n = 0.5n_0$  and temperature  $T = 5$  MeV for several values of the total momentum  $K$ . The solid line gives the total free cross section.  $4\mu_{\text{rel}}$  denotes the position of the effective chemical potential as defined in the text.

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Prof. S.M. Austin  
Editorial Office Physical Review C  
1 Research Road  
Box 1000  
Ridge NY, 11961  
U.S.A.

Dr. T. Alm  
Arbeitsgruppe der MPG  
Theoretische Vielteilchenphysik  
Universität Rostock  
Universitätsplatz 1  
18051 Rostock  
Germany

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